# AUTOMATIC "SEWING TOGETHER" OF REM-IMAGES WITH THE USE OF WALSH DESCRIPTORS

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### Abstract

The article deals with the automatic method of "sewing together" the superimposed REM-images based on the identification of the boundaries of their particularities. The evaluation of a similarity of the form is fulfilled with the use of Walsh descriptors and is used as a key information with a discernment of conjugate areas. The frequency representation of the boundaries allows to determine parameters of optimum transformation of particularities of one image to another. The procedure of identification of an apart from the factor of a similarity of the form, takes into account orientations coherence of the identified areas. The results of a model experiment confirm the effectiveness of a method application.

### 1. Introduction.

Frequently sizes researched with the help of REM exemplars do not allow to use the large magnifications. Therefore the quality of obtained REM-images is lost. That is why the study of such exemplars with the help of REM requires the use of several separate overlapped REM-images, from which afterwards can collect the full scene. Such large-areas images facilitate the interpretation of complicated texture of a sample, as they give the full picture of couplings. As the stacking manually to the one whole even some separate images is labour-consuming operation, we developed an automatic method of "sewing together" of overlapped REM-images, to which the represented work is devoted.

### 2. Segmentation of images.

The preparatory stage of handling the REM-images consists in digital handling supposing their digitization and representation as discrete functions of a grey level. After that one can carry out the segmentation of images on the area corresponding to the structural features. With in the limits of such areas the function of a grey level varies poorly, and to their boundaries there correspond the spasmodic modifications of this function or their derivatives.

It is offered to apply a method of selection of the singularity boundaries using the Marr's operator (Shenk A, 1987). Thus as the boundaries consider zero intersections of a discrete two-dimensional convolution of the function of a grey level I(x,y) with an operator

$$\nabla^2 G(\mathbf{x}, \mathbf{y}) = \left[\frac{\mathbf{x}^2 + \mathbf{y}^2}{\sigma^2} - 2\right] e^{-\frac{C+\mathbf{y}^2}{2\sigma^2}}.$$
 (1)

As the result of an application of this method we obtain the closed curves in the image planes, given by equation

$$C(x, y) = \nabla^2 G(x, y) * I(x, y) = \sum_{i=1}^n \sum_{j=1}^n I(i, j) \cdot \nabla^2 G(x - i, y - j) = 0, x = \overline{1, n}, y = \overline{1, n}.$$
 (2)

The parameter  $w = 2\sqrt{2\sigma}$  characterizes a degree of smoothing the images and is selected empirically for each separate case to supply optimum for a further identification of an amount, sizes and mutual arrangement of the curves.

As C(x, y) is the discreet function of the whole values x, y, satisfying the equation, may not exist. Then on the adjacent pixels the function C(x, y) will accept values of different signs. Let's consider pixel (x, y) as inhering to zero intersection, if  $C(x, y) \le 0$  and for adjacent with him pixel (x', y') the inequality C(x', y') > 0 is fulfilled. The chosen outlines limit the areas of segmentation, which are used for further identification on an overlapped image, definition of parameters of mutual orientation and quantitative estimations residual unjointing.

### 3. Walsh descriptors.

For searching conjugate areas of segmentation on the images it is offered to use as characteristic indications of the form so-called Walsh descriptors of the boundaries of chosen areas.

For a segment [0, T] we shall consider a sequence of sectionally continuous Walsh functions  $\{WAL_n(t) | n \in \mathbb{N}, 0 \le t \le T\}$ , accepting only two value: 1 and -1, and which form an orthogonal system (Гайский В.А., Угупов Н.Д., Ю.П. Корюшкин, 1993). It is known, that any function f(t). integrated on a segment [0, T], can be presented as Walsh series:

$$f(t) = a_0 WAL_0(t) + \sum_{n \ge 1} a_n WAL_n(t),$$
(3)

where the factors of expansion  $\{a_i, i \ge 0\}$  name as Walsh descriptors and calculate under the formula:

$$a_{n} = \frac{1}{T} \int_{0}^{1} f(t) W A L_{n}(t) dt , \ n \ge 0.$$
(4)

In case of discrete function f(t) defined on a segment [0, T], broken on  $N = 2^{k}$  equal parts, for an evaluation of descriptors it is possible to use the following formula:

$$\vec{a} = \frac{1}{N} \left( Wal \right)_{i,j=1}^{N} \vec{f} \,. \tag{5}$$

or in an extended aspect

where  $\bar{a} = (a_0, a_1, \dots, a_{N-1})^t$  - vector-column of expansion factors of function in Walsh series.  $\bar{f} = (f(0), f(T/N), f(2T/N), \dots, f((N-1)T/N))^t$  - vector-column of values of function f(t) in points of a partition of a segment [0, T], and  $(Wal)_{t/t=1}^N$  - matrix appropriate to a set of the first  $N = 2^r$  Walsh functions.

The Walsh descriptors have the following properties:

- 1. As the Walsh functions are mutually orthogonal, the results of comparison of factors of an expansion in a Walsh series are mutually independent.
- 2. The evaluation of factors is easy and includes only operations of addition and subtraction.
- 3. Majority of characteristic features which describe the boundary of function are represented in leading coefficients of Walsh expansion.

Taking into consideration (6),

$$a_0 = \frac{1}{N} \sum_{i=0}^{N-1} f(iT/N), \qquad (7)$$

e.g. factor  $a_0$  is an average value of function f(t) on a segment [0, T]. Similarly it is possible to show, that factor  $a_1$  - generalization of the first derivative of function f(t), and factor  $a_2$  - its second derivative. The consequent factors have not the simple interpretation, but they also can be considered as numerical performances of function f(t) on a segment [0, T].

# 4. Comparison of boundaries of areas of segmentation with the applying of Walsh descriptors

The boundaries of areas of the first image are transferred in the boundaries conjugate with the help of the linear transformation, which is a superposition of parallel transposition, turn and process of scaling. For the definition of the parameters of this transformation we shall present the matched boundaries of areas as function of a distance from a center of masses of curve

$$r(t) = \sqrt{\left(x(t) - x_0\right)^2 + \left(y(t) - y_0\right)^2}, \ t \in [0, T],$$
(8)

where  $x_0, y_0$  = coordinates of a center of masses of a curve, T - length of a curve, t - a natural parameter, x(t), y(t) - corresponding to it coordinates. As an initial point of each curve we shall consider a point, which abscissa is equal to  $x_0$ , and ordinate is least. Thus the position vector of such point with the beginning in a center of masses of a curve is opposite directed to an axes of ordinates. Detour of area we shall realize counter-clockwise. The

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function r(t) can periodically be continued with a phase T, assuming r(t+nT) = r(t),  $t \in [0, T]$ , n - integer.



Fig. 1. Sequential representation of the boundary with the applying of Walsh descriptors.

a) determination of a center of masses and index point of the closed discrete curve;

b) determination of function of a distance of points of a curve from a center of masses;

# c) calculation of Walsh descriptors of function r(t).

Let's consider a modification of function r(t) with elementary linear transformations. It is obvious, that with the parallel transposition of a curve given by a vector  $(\Delta x_{i}, \Delta y)$ , function of the transformed curve  $\mathcal{F}_{max}(t)$  to not vary, that is

$$\widetilde{r}_{loar}(t) = r(t), \ t \in [0, T].$$
(9)

To turn curve on an angle  $\theta$  there will correspond(meet) a shift along a curve  $t_{0}$ , that is

$$\widetilde{r}_{max}(t) = r(t - t_0) = \begin{cases} r(t + (T - t_0)), & 0 \le t \le t_0 \\ r(t - t_0), & t_0 < t \le T \end{cases}$$
(10)

where  $\theta$  and  $t_{a}$  are connected by relation

$$\cos\theta = \frac{(y(0) - \dot{y}_0) \cdot (y(t_0) - y_0)}{r(0) \cdot r(t_0)}.$$
(11)

With a process of scaling, obviously, the equality will be fulfilled

$$\mathcal{P}(t) = \alpha r(t), \ t \in [0, T],$$
 (12)

where  $\alpha$  – relation of scales of the transformed and initial images.

Thus, to resulting transformation of a curve with parameters  $(\Delta x, \Delta y)$ ,  $\theta$  and  $\alpha$ , there will correspond(meet) function  $\tilde{r}(t) = \alpha r(t - t_0)$ .

Let they  $r_1(t)$  and  $r_2(t)$  - functions of a distance(span) from a center of masses of compared curves obtained on the formula (8) Let's define parameters  $\theta$  and  $\alpha$  of transformation transforming the first curve in to the second, assuming, that

$$r_{2}(t) = \alpha r_{1}(t - t_{0})$$
(13)

For this purpose we shall calculate the first  $N = 2^i$  functions  $r_1(t)$  and  $r_2(t)$  of Walsh descriptors under the formula (6). Taking into account (12) and the periodicity of functions  $r_1(t)$  and  $r_2(t)$ , it is possible to determine  $\alpha$  under the formula

$$\alpha = \frac{a_0^2}{a_0^1} \tag{14}$$

where  $a_{1}^{t}, a_{2}^{t}$  are - zero Walsh descriptors of functions  $r_{1}(t)$  and  $r_{2}(t)$  accordingly. Sorting out the values of  $t_{0}$  from the interval [0, T), we shall define magnitude

$$C_{12} = \min_{a_1+a_2} \sum_{t=0}^{N-1} (\alpha a_t^1 - a_t^2(t_0))^2, \qquad (15)$$

where  $a_i^t, a_i^t(t_n)$  are the Walsh i-descriptors of functions  $r_1(t)$  and  $r_2(t+t_0)$  accordingly. This magnitude is a measure of identity between a pair of compared closed curves. It characterizes distinction of the form between the optimum transformed curve of the first image and compared to it of a curve of the second image. The value  $t_0$ , with which the minimum of a

sum  $\sum_{i=0}^{N-1} (\alpha a_i^1 - a_i^2(t_0))^2$  is reached, corresponds to a best value of a parameter  $\theta$ , which is determined under the formula (11).

Thus, for a pair (i, j) of compared areas we obtain a measure of identity  $C_{i}$ , and parameters of optimum transformation  $\theta_{i}$  and  $\alpha_{i}$  of area i to area j.

# 5. Goal function of identification.

If there arc some areas of segmentation of the similar form in a common part of overlapped images, the identification of their boundaries based only on the criterions of a similarity of the form, is unreliable. For the amplification of conditions of an identification, it is offered to use the restrictions to combined share orientation of pairs of compared areas. Let's emanate from the supposition, that the boundaries of conjugate areas are not only similar to the form, but also the parameters of mutual orientation of conjugate pairs are close among themselves.

Taking into account these factors, we shall make the goal function of identification, which would be for the evaluation of quality of an identification of segmentation areas of two images and reached a minimum on an optimum condition of identification. Let is present m Numbered areas of segmentation on the first image and n - on the second. The condition of identification can be presented as aspect of a matrix  $V = (v_n)_{n,n=1}^{m,n}$  of a size  $m \times n$ , which elements are zeros and units. Let's  $v_n = 1$  assume in only case when, the i-area of the first image is identified to j-area of the second. The first addend of the goal function will reflect a similarity of the form of the boundaries of the identified areas:

$$f_{1}(V) = \sum_{i=1}^{m} \sum_{j=1}^{n} v_{ij} C_{ij}$$
(16)

The second addend respond to a coherence of orientational parameters of all identified pairs of areas among themselves:

$$f_{2}(V) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{j=1}^{n} v_{ij} \cdot W_{ijkk} \cdot v_{ij}$$
(17)

where  $W_{(i_l,k_l)} = \sqrt{(\alpha_{i_l} - \alpha_{i_l})^2 + (\theta_{i_l} - \theta_{i_l})^2}$  - magnitude describing a coherence parameters of orientation of identified pairs (i, j) and (k, l). The goal function of identification now can be noted as

$$F(V) = c_1 f_1(V) + c_2 f_2(V)$$
(18)

where  $c_1, c_2$  weight factors, which are selected for searching of optimum solution of the task. The minimization of the goal function can be executed with the help of neural network (Yi-Hsing Tseng, Jin-John Tzen, Kei-Pay Tang, Shin-Hung Lin, 1997), or in case of a small amount of areas of segmentation by simples exhaustive search of all possible matrixes V of a condition of identification. As a result we obtain optimum identification of areas of segmentation.

"Sewing together" of images is spent, fulfilling parallel transposition of the second image concerning the first, given by a vector  $(x'_0 - x''_0, y'_0 - y''_0)$ , where  $(x'_n, y'_n)$ ,  $(x''_0, y''_0)$  centers of masses of a pair (i, j) of the identified curve of the first and the second images accordingly,  $C_n$  magnitude is least for a set of all identified pairs. Then the turn of the second image of an angle  $\theta_n$  clockwise of rather concurrent center of masses in a point  $(x'_0, y'_0)$  is fulfilled and at last the process of scaling of the second image with a parameter  $\alpha_n$  is fulfilled.

The quality of "sewing together" of images can be estimated by the magnitudes average quadratic deviations of angle  $\sigma_{\theta}$  of a turn and parameter of process of scaling  $\sigma_{\alpha}$  to all identified pairs of areas of images.

### 6. Results of experiment.

We carried "sewing together" of four REM-images of 1600-multiple magnification obtained with the help of electronic microscope Hitachi S800. Chosen with the help of operator Mappa (1) boundaries of the singularities of these images are represented in Figure 2. In Figure 3 the result of automatic "sewing together" of these images is represented.

### Conclusions.

7.

The method of automatic "sewing together" partially overlapped REM-images, assumes identification of the boundaries texture of singularities submitted in the frequency form by Walsh descriptors. Such representation of the boundaries allows to determine the parameters of optimum transformation of singularities of one image to another other. The procedure of identification of an apart from the factor of a similarity of the form, takes into account orientational coherence of the identified areas, that increases an exactitude of "sewing together". The algorithm of a method is simples for a realization as the software. The outcomes of a model experiment confirm the effectiveness of application of this method.

### LITERATURE

1. Гайский В.А., Угупов Н.Д.: Ю.П. Корюшкин. Применение функций Уолша в системах автоматизации научных исследований. –К.:Наук. Думка, 1993. –212с.

Melnyk V.M.,, Voloshyn V.U., Voloshyna T.V. - "Automatic "sewing together" of REM-images 5-25 with the use of Walsh descriptors"

- 2. Shenk A. Eine Methode zur Besstimmung von Signalanderungen und Anwendungsbeispiele //Z.f.v. 1987. 4. -S.172-178.
- Yi-Hsing Tseng, Jin-John Tzen, Kei-Pay Tang, Shin-Hung Lin. Image-to-Image Registration by Matching Area Features Using Fourier Descriptors and Neural Networks // Photogrammetric Engeneering & Remote Sensing - 1997. - 63, No.8, - pp.975-983.



Fig. 2. The chosen boundaries of singularities of REM-images.



Fig. 3. An outcome of "sewing together" of REM-images, obtained with the help of electronic microscope with magnification 1600<sup>x</sup>.

